

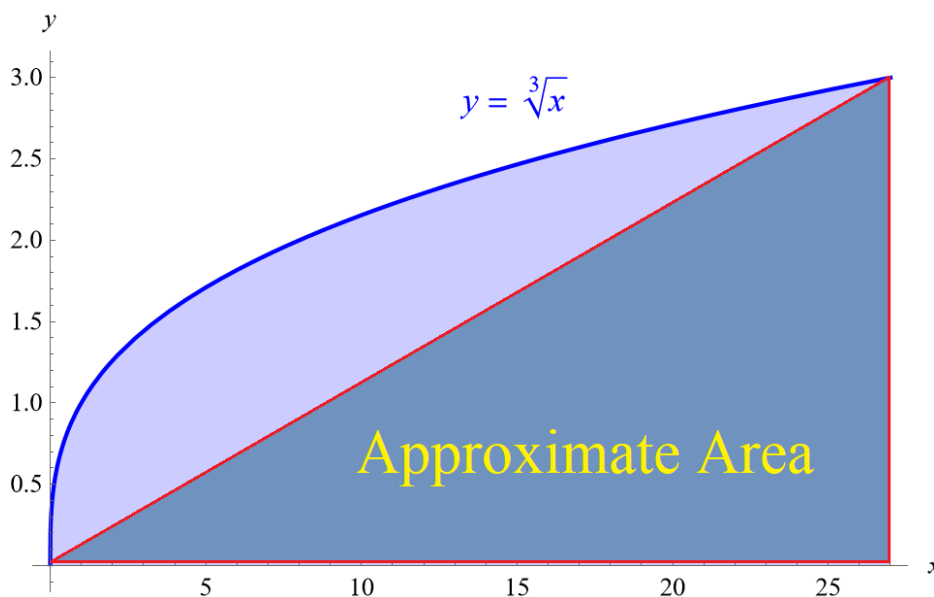
Exercise 49

Use a graph to give a rough estimate of the area of the region that lies beneath the given curve. Then find the exact area.

$$y = \sqrt[3]{x}, \quad 0 \leq x \leq 27$$

Solution

Below is a graph of the given function.



The shaded area under the curve is loosely the area of a triangle with base 27 and height 3.

$$\text{Area} \approx \frac{1}{2}(27)(3) = \frac{81}{2} = 40.5$$

Calculate the shaded area exactly by evaluating the integral of the function from $x = 0$ to $x = 27$.

$$\begin{aligned} \text{Area} &= \int_0^{27} \sqrt[3]{x} \, dx \\ &= \int_0^{27} x^{1/3} \, dx \\ &= \left. \frac{x^{4/3}}{\frac{4}{3}} \right|_0^{27} \\ &= \frac{3}{4}(27^{4/3} - 0^{4/3}) \\ &= \frac{243}{4} \approx 60.75 \end{aligned}$$